

Solving Quadratic Problems

This booklet belongs to: _____

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1.		Pg.	
2.		Pg.	
3.		Pg.	
4.		Pg.	
5.		Pg.	
6.		Pg.	
7.		Pg.	
8.		Pg.	
9.		Pg.	
10.		Pg.	
11.		Pg.	
12.		Pg.	
13.		REVIEW	
14.		TEST	

Your teacher has important instructions for you to write down below.

The answers to challenges are usually on the next page.

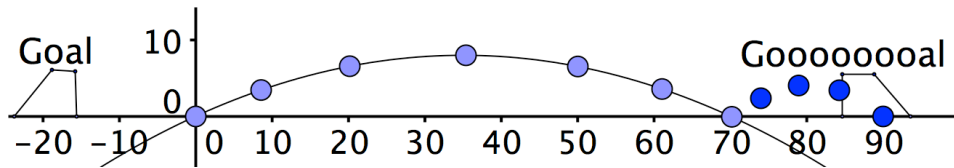
Definitions

		Examples
Completing the square	This technique is used to turn a quadratic function in standard form, $y = ax^2 + bx + c$, into vertex form $y = a(x - p)^2 + q$	
How long until an object hits the ground.	This is word problem question. It is asking for the x-intercept. This can be found by setting $y=0$ and solving for x.	
Parabola	The symmetrical graph of a quadratic function.	
Quadratic Function	A polynomial function of degree 2.	
Standard form	$y = ax^2 + bx + c$	
Vertex form	$y = a(x - p)^2 + q$	
Y-intercept	The coordinate where the curve crosses the y-axis. This can be found algebraically by setting $x=0$ and solving for y.	
X-intercept	The coordinate where the curve crosses the x-axis. This can be found algebraically by setting $y=0$ and solving for x.	
Solution to a quadratic equation	Values that satisfy both sides of an equation	The solutions to $(x + 2)(x - 5) = 0$ are $x = -2$ and $x = 5$
Roots of a quadratic equation	The roots of an equation are the solutions of the equation.	The roots to $(x + 2)(x - 5) = 0$ are $x = -2$ and $x = 5$
Zeros of a quadratic function	The zeros of a quadratic function are the x-intercepts.	
Quadratic Equation	An equation with degree two.	Example: $x^2 + 3x - 7 = 0$ Non-example: $x^2 + 3x^3 - 7 = 0$
Extraneous root	An extraneous root is a false solution. It may satisfy the equation but there it does not meet initial restrictions.	In an area problem, the side lengths are found to be -2 and 7. -2 is an extraneous root since side lengths of negative length do not make sense.
Quadratic Formula	The quadratic formula states that if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$.	
Discriminant	The discriminant is an expression found under the square root sign of the quadratic formula. $b^2 - 4ac$	
Two distinct real roots		
One distinct real roots		
No real roots		

Solve by Graphing

Parabolic Goal Kicks

There is a great clip on YouTube where a goalie actually scores a goal with a huge goal kick. The graph below models the flight of the ball after it is kicked. The ball follows the pathway $v = -0.03d^2 - 2.1d$ where v is the vertical height in meters and d is the horizontal distance in meters.



** <http://www.youtube.com/watch?v=tdqpZU8i9rQ> ** <http://www.youtube.com/watch?v=05VhDBSQ8lQ> **

1. Analyze the graph to determine the total horizontal distance travelled by ball before it bounces for the first time.
2. What key points on the graph did you use to answer this question?
3. What is the y coordinate at the x-intercepts?

In this section you will learn how to solve quadratic equations by graphing.

Challenge: Solve $x^2 - 6x + 5 = 0$ by graphing.

Read the following carefully and answer the associated questions.

Solve $x^2 - 6x + 5 = 0$ by substituting 0 with y .

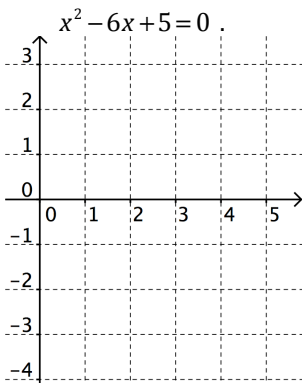
$$0 = y = x^2 - 6x + 5$$

Complete the square and write the equation in vertex form.

$$y = (x^2 - 6x + 9) - 9 + 5$$

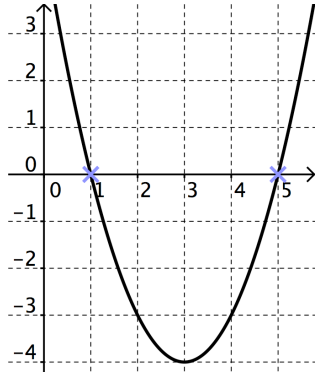
$$y = (x - 3)^2 - 4$$

4. Graph $y = x^2 - 6x + 5$ to solve

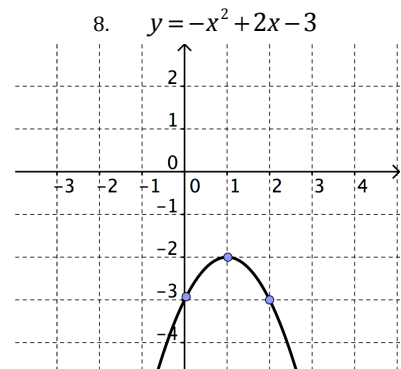
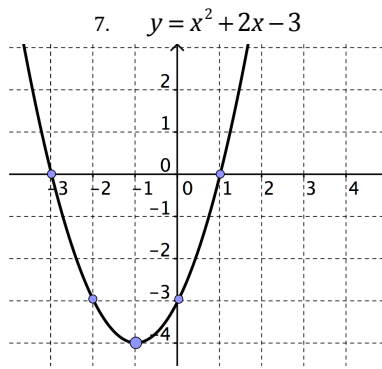
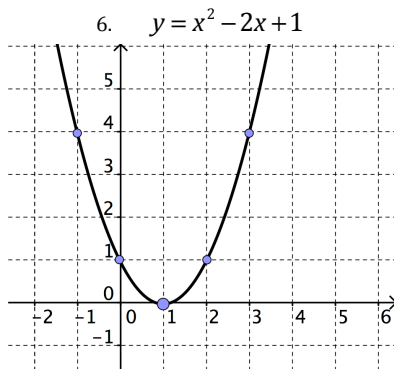


5. What coordinates on the graph make the equation, $x^2 - 6x + 5 = 0$, equal zero?

Solve Quadratic Equations by Graphing

<p>Quadratic function $y = ax^2 + bx + c$</p> <p>The zeros are the x-intercepts of a quadratic function.</p> <p>For example: The x-intercepts or zeros of $y = x^2 - 6x + 5$ are 5 and 1.</p>	<p>$y = x^2 - 6x + 5$</p> 	<p>Quadratic equation $ax^2 + bx + c = 0$</p> <p>Roots are the solutions to a quadratic equation.</p> <p>For example: The solutions or roots of $x^2 - 6x + 5 = 0$ are 5 and 1.</p>
<p>You can solve a quadratic equation in the form $ax^2 + bx + c = 0$ by graphing the corresponding quadratic function $y = ax^2 + bx + c$.</p>		

What are the x-intercepts of each parabola?

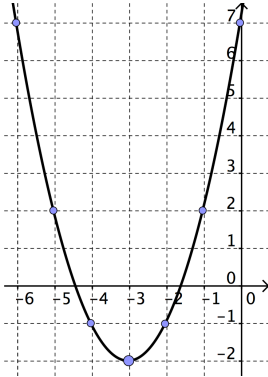


9. What is the value of y at the x-intercept?

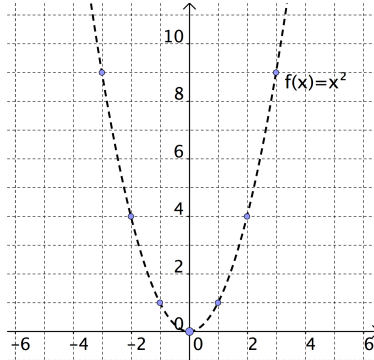
10. Explain why the zeros of $y = ax^2 + bx + c$ and the roots of $ax^2 + bx + c = 0$ are the same.

How many unique real roots does each graph have?

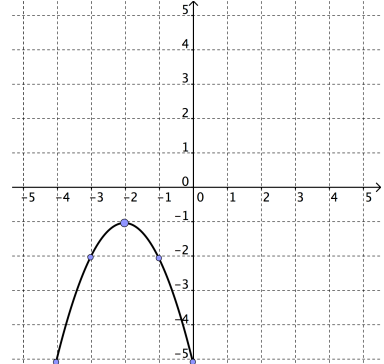
11.



12.



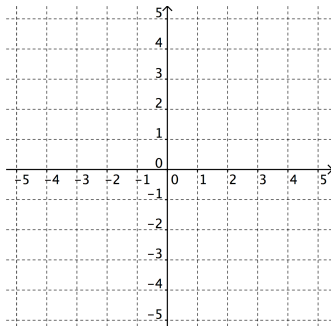
13.



Determine the x-intercepts of the function and the roots of the equations by graphing.

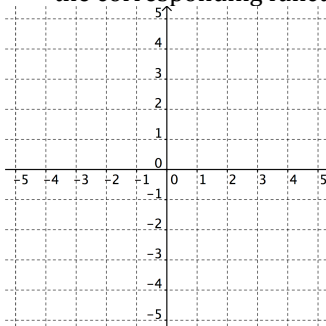
14. Determine the x-intercept(s)

of $y = (x+2)^2 - 1$.



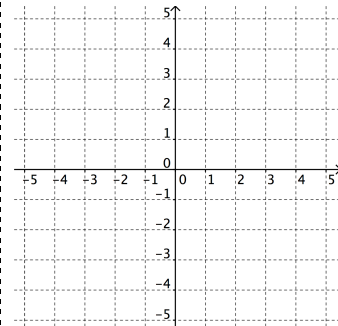
15. Determine the roots of

$0 = -(x-3)^2 + 4$ by graphing the corresponding function.

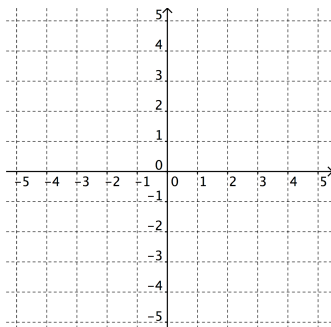


16. Determine the x-intercept(s)

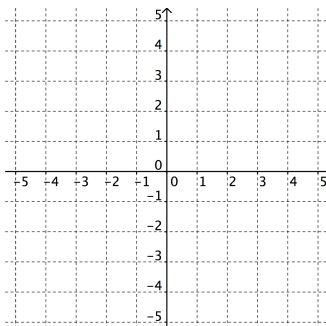
of $y = (x-3)^2$.



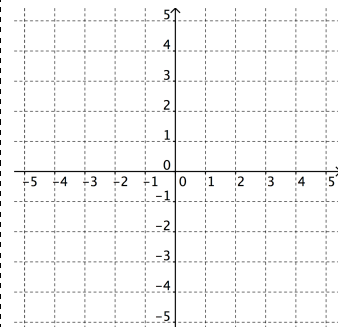
17. Determine the roots of $(x+1)^2 - 4 = 0$ by graphing the corresponding function.



18. Determine the x-intercept(s) of $y = -(x-2)^2$.



19. Determine the roots of $(x-2)^2 + 1 = 0$ by graphing the corresponding function.



Determine the solution of each equation by graphing.

20. Solve $x^2 + 6x + 8 = 0$.

Solution:

Write the corresponding quadratic function:

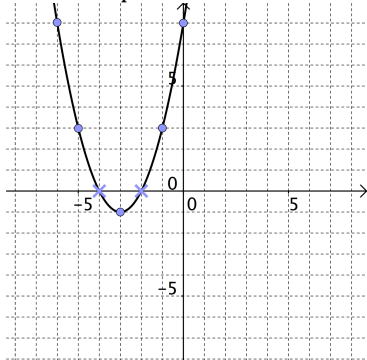
$$y = x^2 + 6x + 8$$

Complete the square

$$y = (x^2 + 6x + 9) - 9 + 8$$

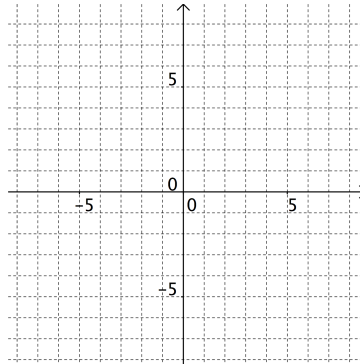
$$y = (x + 3)^2 - 1$$

Graph the function

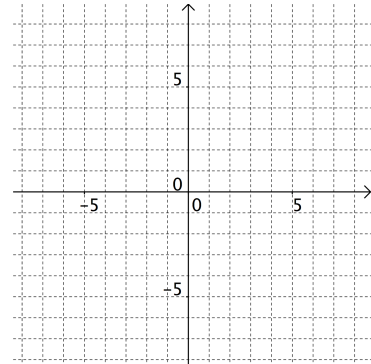


The x-intercepts are -2 and -4. The roots of the equation are -2 and -4.

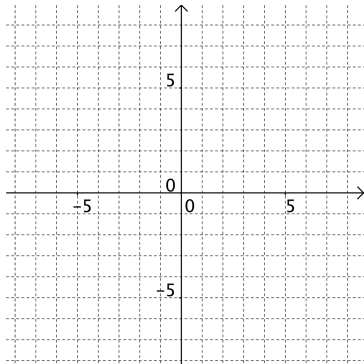
21. Solve $0 = x^2 + 4x$.



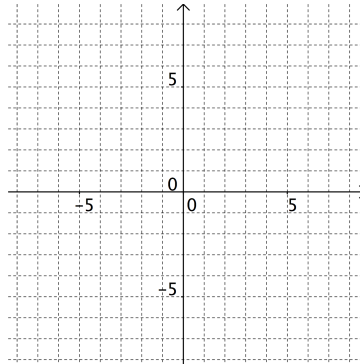
22. Solve $2x^2 + 16x + 24 = 0$.



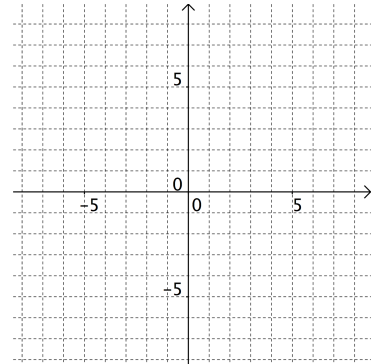
23. Solve $0 = \frac{1}{2}x^2 - 4x + 6$.



24. Solve $2x^2 - 16x + 32 = 0$.



25. Solve $0 = -x^2 - 5x$.

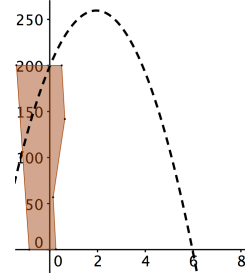


Use the graph to answer the related questions.

26. A piece of lava rock is thrown off of a 200-foot cliff. The height of the rock follows the parabolic pathway
 $h = 16t^2 + 62t + 199$ where h is height in meters and t is time in seconds.

How long does it take for the rock to hit the ground?

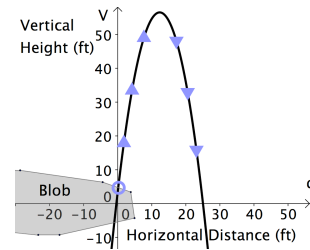
Which coordinates did you use to answer the question above?



Use the graph to answer the related questions.

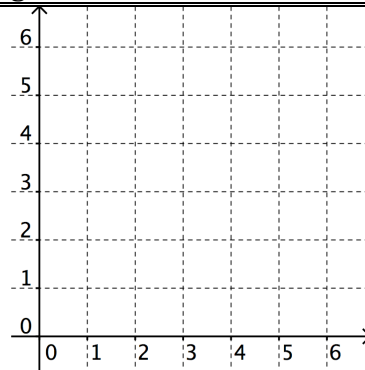
27. A person is projected off the inflatable blob. Their pathway through the air is given by the function $v = -0.35d + 8.6d + 3$ where v is the person's vertical height in feet and d is horizontal distance in feet.

Use the graph to approximate the horizontal distance travelled by the person at the exact moment he lands in the water.

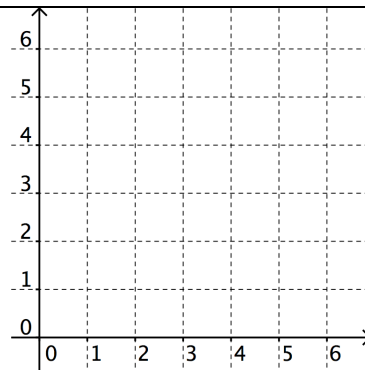


Solve each question by graphing.

28. A toad jumps from a tree stump to the ground below. The toad's vertical height follows the pathway
 $h = -d^2 + 2d + 3$ where h is height in feet and d is horizontal distance in feet. Determine the horizontal distance travelled by the toad when it lands.



29. A cat jumps from a stationary trying to catch a moth. The cat's height relative to horizontal distance can be modeled by the function;
 $h = -\frac{1}{2}d^2 + 2d$, where h is height in meters and d is distance in meters. Determine the total horizontal distance when the cat lands on the ground.

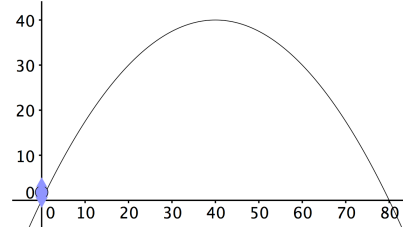


Use the graph to answer the related questions.

30. A goal kick in soccer is kicked from the ground. The height of the ball can be modeled by this quadratic function

$$h = -0.025d^2 + 2d$$
 where h is height in feet and d is distance in feet.

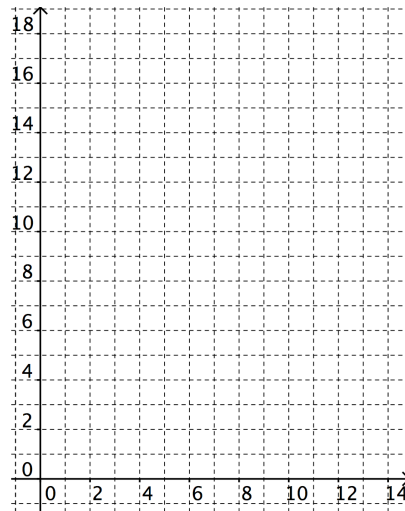
Which coordinates do you need to know to determine the total horizontal distance travelled by the ball before it bounces?



Solve each question by graphing.

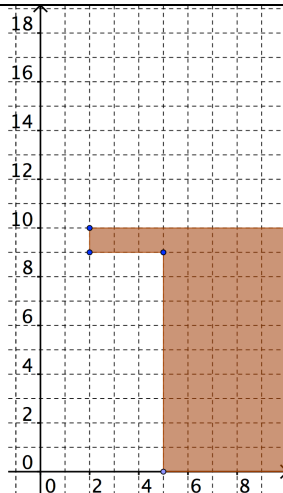
31. Jordan is standing in his tree fort and throws a toy soldier with a parachute into the air. The soldier's initial height was 10feet and once tossed follows a pathway

$$h = -\frac{1}{2}t^2 + 4t + 10$$
 where h is height in meters and t is time in seconds. How long does it take for the toy soldier to land on the ground?



32. A cricket jumps from the ground to a ledge that is 10 inches tall. The cricket's height can be modeled by the quadratic function

$$h = -2d^2 + 12d$$
 where h is height in inches and d is distance in inches. Determine the horizontal distance travelled when the cricket lands?



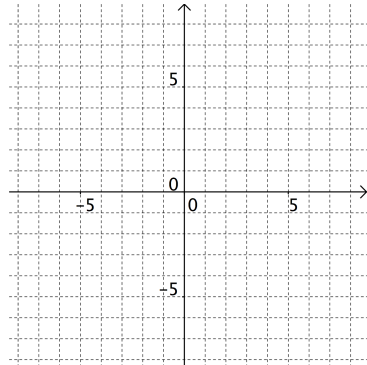
Solve Quadratic equations by Factoring

2-Minute Challenge:

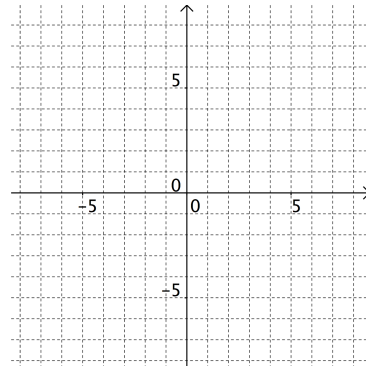
- Solve each equation by graphing
- All of the following questions have exact solutions

Solve each of the following by graphing if possible.

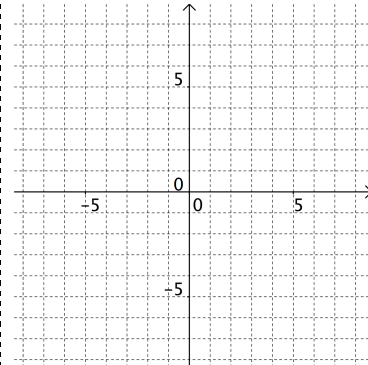
33. Solve $(x-2)^2 - 4 = 0$.



34. Solve $\left(x - \frac{1}{2}\right)^2 - 2.25 = 0$.



35. $4(x+1)^2 - 16 = 0$



 36. Were you able to solve the problems above? If not which one(s) and how come?

From yesterday's lesson we know that if we know how to graph, solving by graphing is relatively straightforward. However, we can see above that solving by graphing has some limitations. These limitations are exposed when the vertices are not exact or the vertices do not fit on the grid. These conditions make it difficult to graph the function accurately, which in turn makes it difficult to find the solutions. In today's lesson we will see how factoring can help us solve problems that graphing cannot.

Solving Quadratic Equations by Factoring

Let's do a quick review of factoring before trying to solve quadratic equations by factoring.

Factoring Review.

37. Factor $x^2 + 10x + 16$

38. Factor $5x^2 + 17x + 6$.*

39. $2x^2 + 11x + 5$

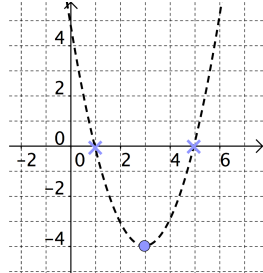
* Three different methods. Use any method.

<p>Factor $5x^2 + 17x + 6$.</p> <p>Factor $5x^2 + 17x + 6$ $(ax + b)(cx + d)$ $a \times c = 5$, Options $\rightarrow 1$ & 5. $b \times d = 6$, Options $\rightarrow (\pm 1 \& \pm 6)$ or $(\pm 2 \& \pm 3)$* Possible options**: $(5x + 1)(1x + 6) = 5x^2 + \cancel{31x} + 6$ $(5x + 6)(1x + 1) = 5x^2 + \cancel{11x} + 6$ $(5x + 3)(1x + 2) = 5x^2 + \cancel{13x} + 6$ $(5x + 2)(1x + 3) = 5x^2 + 17x + 6$</p>	<p>Factor $5x^2 + 17x + 6$. Find two numbers that:</p> <ul style="list-style-type: none"> • Multiply to $5 \times 6 = 30$ • Add to 17 <div style="text-align: center; margin: 10px 0;"> <table border="1" style="display: inline-table; border-collapse: collapse;"> <tr> <td style="padding: 5px;">15</td> <td style="padding: 5px;">×</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">30</td> </tr> <tr> <td style="padding: 5px;">+</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> </table> <p>17</p> </div> <p>$5x^2 + [17x] + 6$ Rewrite 17x as 15x+2x. $5x^2 + [15x + 2x] + 6$ $5x^2 + 15x + 2x + 6$ Group and factor out GCF. $(5x^2 + 15x) + (2x + 6)$ $5x(x + 3) + 2(x + 3)$ Factor out x+3. $(5x + 2)(x + 3)$</p>	15	×	2	30	+				2				<p>Factor $6x^2 + 13x + 6$. Find two numbers that:</p> <ul style="list-style-type: none"> • Multiply to $6 \times 6 = 36$ • Add to 13 <div style="text-align: center; margin: 10px 0;"> <table border="1" style="display: inline-table; border-collapse: collapse;"> <tr> <td style="padding: 5px;">9</td> <td style="padding: 5px;">×</td> <td style="padding: 5px;">4</td> <td style="padding: 5px;">36</td> </tr> <tr> <td style="padding: 5px;">+</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">4</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> </table> <p>13</p> </div> <p>$6x^2 + 13x + 6$ Place the x^2 coefficient in front of each x. $\left(\frac{6x + ?}{??}\right)\left(\frac{6x + ?}{??}\right)$ Place the 4 and the 9 in for "?" $\left(\frac{6x + 4}{??}\right)\left(\frac{6x + 9}{??}\right)$ Divide the product by the x^2 coefficient. (6) $\left(\frac{6x + 4}{2}\right)\left(\frac{6x + 9}{3}\right)$ Simplify $(3x + 2)(2x + 3)$</p>	9	×	4	36	+				4			
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Solve.

40. What are the x-intercepts of the graph below?

$x = \underline{\hspace{1cm}}$ & $x = \underline{\hspace{1cm}}$

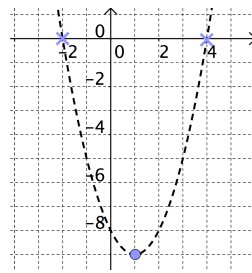


41. Each of the following equivalent equations represents the above graph. Which equation below is it easiest to know the x-intercepts.

- A. $y = (x-3)^2 - 4$
- B. $y = (x^2 - 6x + 9) - 4$
- C. $y = x^2 - 6x + 5$
- D. $y = (x-1)(x-5)$.

42. What are the x-intercepts of the graph below?

$x = \underline{\hspace{1cm}}$ & $x = \underline{\hspace{1cm}}$

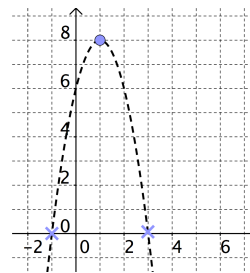


43. Each of the following equivalent equations represents the above graph. Which equation below is it easiest to know the x-intercepts.

- A. $y = (x-1)^2 - 9$
- B. $y = (x^2 - 2x + 1) - 9$
- C. $y = x^2 - 2x - 8$
- D. $y = (x-4)(x+2)$.

44. What are the x-intercepts of the graph below?

$x = \underline{\hspace{1cm}}$ & $x = \underline{\hspace{1cm}}$



45. Each of the following equivalent equations represents the above graph. Which equation below is it easiest to know the x-intercepts.

- A. $y = -2(x-1)^2 + 8$
- B. $y = (-2x^2 + 4x - 2) + 8$
- C. $y = -2x^2 + 4x + 8$
- D. $y = -2(x^2 - 2x - 4)$
- E. $y = -2(x-3)(x+1)$.

46. Explain why finding the x-intercepts of the equation $y = (x-1)(x-5)$ has the same answer as solving the equation $0 = (x-1)(x-5)$

Challenge:

47. Determine the x-intercepts
 $y = x(x+5)$.

48. Solve $0 = (2x+1)$.

49. Solve: What values of x make the right side equal to zero?
 $0 = (2x+1)(x+5)$

<p>50. Determine the x-intercepts $y = x(x+5)$.</p> <p>Solution. $y = x(x+5)$ $y=0$ at the x-intercepts $0 = x(x+5)$ The solutions are $x=0$ or $x=-5$</p>	<p>51. Solve $0 = (2x+3)(x-2)$.</p>	<p>52. State the x-intercepts of $y = 5x(x-2)$</p>
<p>53. Solve $0 = (2x+1)(x+5)$.</p> <p>Solution. For the right side to equal 0 either $(2x+1)=0$ $2x = -1$ or $(x+5)=0$ $x = -\frac{1}{2}$ $x = -5$</p>	<p>54. $0 = 7(x+1)(2x-1)$</p>	<p>55. $(5x+1)(3x-8) = 0$</p>

Write a quadratic equation that has the following x-intercepts in the form $y = (ax-b)(cx-d)$

<p>56. $(5,0) \& (-2,0)$</p>	<p>57. $(-3,0) \& (-1,0)$</p>	<p>58. $(\frac{2}{5}, 0) \& (4,0)$</p>
<p>59. $(-\frac{1}{2}, 0) \& (\frac{1}{3}, 0)$</p>	<p>60. $(-1,0) \& (0,0)$</p>	<p>61. $(0,0) \& (-17,0)$</p>

Challenge:

<p>62. Solve $x^2 + 6x - 16 = 0$</p>	<p>63. Solve $5m^2 + 150 = 55m$.</p>	<p>64. Solve: $0 = 3m^2 + 16m + 5$</p>
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Solve each equation.

<p>65. $x^2 + 6x - 16 = 0$ Solution: $x^2 + 6x - 16 = 0$ Factor and solve. $(x+8)(x-2) = 0$ $x = -8$ or $x = 2$</p>	<p>66. $0 = x^2 + 2x - 24$</p>	<p>67. $0 = x^2 - 2x$</p>
<p>68. $5m^2 + 150 = 55m$ Solution: $5m^2 + 150 = 55m$ Rearrange $5m^2 - 55m + 150 = 0$ Factor out GCF $5(m^2 - 11m + 30) = 0$ Factor. $5(m-5)(m-6) = 0$ $m = 5$ or $m = 6$</p>	<p>69. $x^2 - 9x = 36$</p>	<p>70. $-4m^2 = 16m - 180$</p>
<p>71. $0 = 3m^2 + 16m + 5$ Solution. $0 = 3m^2 + 16m + 5$ Factor by any method. $0 = (3m + \underline{\quad})(1m + \underline{\quad})$ $0 = (3m + 1)(1m + 5)$ $3m + 1 = 0$ $3m = -1$ or $m + 5 = 0$ $m = -\frac{1}{3}$ or $m = -5$</p>	<p>72. $0 = 6m^2 + 17m + 5$</p>	<p>73. $12m^2 + 30 = -66m$</p>

Solve each equation.

74. $0 = x^2 - 18x + 77$

75. $0 = 2m^2 - 5m + 3$

76. $0 = 2m^2 + 14m + 20$

77. $30x^2 - 40x = 70$

78. $x^2 = 46x - 45$

79. $0 = -15m^2 + 10m^3 - 25m$

Challenge:

80. Solve $0 = 36x^2 - 25$

81. Solve. $0 = 49 - (3x + 1)^2$

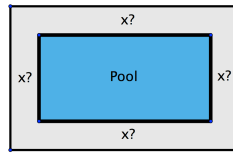
82. Solve: $(x + 1)^2 + 5(x + 1) + 6 = 0$

Solve each equation.

<p>83. $0 = 36x^2 - 25$ Solution: $0 = 36x^2 - 25$ $0 = (6x+5)(6x-5)$ $6m+5=0$ $6m-5=0$ $6m=-5$ or $6m=5$ $m = -\frac{5}{6}$ $m = \frac{5}{6}$</p>	<p>84. $0 = 121x^2 - 49$</p>	<p>85. $0 = 1 - \frac{81}{100}x^2$</p>
<p>86. $0 = 49 - (3x+1)^2$ Solution. $0 = 49 - (3x+1)^2$ This is the form $A^2 - B^2$ Factor $0 = (A-B)(A+B)$ $0 = (7 + [3x+1])(7 - [3x+1])$ Simplify in each bracket & solve. $0 = (3x+8)(-3x+6)$ $3x+8=0$ $-3x+6=0$ $3x=-8$ or $-3x=-6$ $x = -\frac{8}{3}$ $x = \frac{6}{3} = 2$</p>	<p>87. $0 = (2x-1)^2 - 1$</p>	<p>88. $0 = (2x+1)^2 - 4(x-2)^2$</p>
<p>89. $(x+1)^2 + 5(x+1) + 6 = 0$ Solution. $(x+1)^2 + 5(x+1) + 6 = 0$ This is in the form $A^2 + 5A + 6$ Factor $(A+2)(A+3) = 0$ $([x+1]+2)([x+1]+3) = 0$ Simplify and solve. $(x+3)(x+4) = 0$ $x = -3$ or $x = -4$</p>	<p>90. $0 = (x-5)^2 - 7(x-5) - 30$</p>	<p>91. $0 = (2x+1)^2 - 19(2x+1) + 90$</p>

Challenge:

92. A swimming pool, measuring 10m by 5m is to have a walkway/deck built around it of uniform width. The total area of the walkway and pool together is to be 126m^2 . Set up and solve a quadratic equation to determine the width of the deck.



93. John releases a paper airplane from a height of 6 feet. It follows a parabolic path with $h = -t^2 + t + 6$, where h is height in feet, and t is time in seconds. Determine how long it takes for the plane to hit the ground.

Solve each problem.

94. A walkway of uniform width is to be built around a swimming pool, measuring 10m by 5m. The total area of the walkway and deck together is to be 126m^2 . Set up and solve a quadratic equation to determine the width of the deck.

Solution:

$$\text{Area} = lw$$

$$A = 126\text{m}^2$$

$$l = 10\text{m} + 2x$$

$$w = 5\text{m} + 2x$$

$$126 = (10 + 2x)(5 + 2x)$$

$$126 = 50 + 20x + 10x + 4x^2$$

$$0 = 4x^2 + 30x - 76 \rightarrow 0 = 2x^2 + 15x - 38$$

The factors of 2 are $\pm 1, \pm 2$

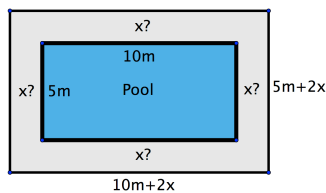
The factors of -38 are $\pm 1, \pm 2, \pm 19, \pm 38$

Factor by Guess and check

$$(2x + 19)(x - 2) = 2x^2 + 15x - 38$$

$$x = \frac{-19}{2} \text{ or } x = 2 \text{ The width is 2.}$$

$$\text{Check } (10 + 2(2))(5 + 2(2)) = 14 \times 9 = 126$$



95. A photograph 8 cm by 11 cm will have a frame with uniform thickness. The combined area of the frame and photograph will be 180cm^2 . Algebraically determine the outside dimensions of the frame.

Solve each problem.

96. John releases a paper airplane from a height of 6 feet. It follows a parabolic path with $h = -t^2 + t + 6$, where h is height in feet, and t is time in seconds. Determine how long it takes for the plane to hit the ground.

Solution:

The plane hits the ground when $h=0$

$$0 = -t^2 + t + 6$$

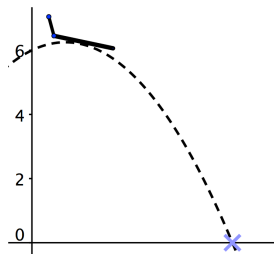
Find the t -intercepts.

$$0 = -(t^2 - t - 6)$$

$$0 = -(t - 3)(t + 2)$$

$$t = \cancel{-2} \text{ or } t = 3$$

The t -intercept is 3. It takes 3 seconds for the plane to hit the ground.



97. An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height h at time t seconds after launch is

$$h = -4.9t^2 + 19.6t + 58.8, \text{ where } h \text{ is in meters. How long until the object hits the ground?}$$

98. A rectangular garden has a perimeter of 140m and an area of 1200m². Determine a quadratic equation that models this situation.

99. Determine the dimensions of the garden.

100. A baseball player hits a ball into the air. The ball's height above the ground, in feet, t seconds after being hit, is approximated by $h(t) = -5t^2 + 13t + 1$. **How long was the ball in the air for if it is caught at a height of 9 feet?**

Solve each equation.

$$101. 0 = 25 - x^2$$

$$102. 0 = x^2 - 36x$$

$$103. 0 = 1 - \frac{1}{64}x^2$$

$$104. 0 = (x+7)^2 - 81$$

$$105. 0 = 3(2x)^2 + 16(2x) + 5$$

$$106. (x+1)^2 - 100(x+4)^2 = 0$$

The Three-Minute Challenge

The Three-Minute Challenge:

- Solve 3 questions in fewer than three minutes.
- All of the problems in the 3-minute challenge have solutions.
- All of the problems in the 3-minute can be solved in less than 3 minutes.
- You may work with in small groups or independently.
- Do not begin until you are instructed to do so.
- Good luck.

107. Solve $x^2 + 8x + 12 = 0$.

108. Solve $x^2 - 40x - 3696 = 0$

109. Solve $x^2 + 8x + 11 = 0$.

Round your answers to the nearest tenth.

110. Were you able to solve the problems above? If not which one(s) and how come?

There are many ways to solve problems. To this point we have learned how to solve equations by graphing and by factoring. We learned that solving by graphing has advantages and limitations. In a similar way solving by factoring also has advantages and limitations as is seen clearly in the examples above. In the next couple of lessons we will learn new techniques that allow us to solve problems that solving by factoring cannot.

Solve by completing the Square

Review: Complete the square.

111. $y = 2x^2 + 12x + 7$	112. $y = x^2 + 10x + 20$	113. $y = 10x^2 + 20x + 3$
$y = (2x^2 + 12x) + 7$		
$y = 2(x^2 + 6x + [0]) + 7$		
$y = 2(x^2 + 6x + [3^2 - 3^2]) + 7$		
$y = 2((x^2 + 6x + 9) - 9) + 7$		
$y = 2((x + 3)^2 - 9) + 7$		
$y = 2(x + 3)^2 + 2(-9) + 7$		
$y = 2(x + 3)^2 - 18 + 7$		
$y = 2(x + 3)^2 - 11$		

Challenge: Round your answer to 3 decimals where appropriate.

114. $x^2 = 4$

115. $(x + 7)^2 = 4$

116. $2(x + 3)^2 = 52$

Solve for x where possible.

117. $x^2 = 4$

Solution:

Solution #1

$x^2 = 4$

Square root
both sides

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

Solution #2

$x^2 = 4$

$x^2 - 4 = 0$

Factor

$(x + 2)(x - 2) = 0$

$x = \pm 2$

$x = 2$ or -2

Confirm

$2^2 = 4, (-2)^2 = 4$

118. $x^2 = 25$

119. $x^2 = -25$

Solve for x.

120. $x^2 = 26$

121. $x^2 = -9$

122. $x^2 = 24$

123. $5x^2 + 6 = 16$

124. $\frac{2}{3}x^2 - 1 = 7$

125. $9x^2 + 14 = -4$

Solve for x where possible. Round your answer to the nearest tenth where appropriate.

126. $(x+7)^2 = 4$

Square root both sides.

$$\sqrt{(x+7)^2} = \pm\sqrt{4}$$

$$(x+7) = \pm 2$$

Isolate x.

$$x = \pm 2 - 7$$

$$x = -5, x = -9$$

Confirm you answer.

127. $(x-5)^2 = 25$

128. $2(x+1)^2 = 98$

129. $2(x+3)^2 = 52$

Divide both sides by 2.

$$(x+3)^2 = 26$$

Square root both sides.

$$\sqrt{(x+3)^2} = \pm\sqrt{26}$$

$$(x+3) = \pm\sqrt{26}$$

Isolate x.

$$x = \pm\sqrt{26} - 3$$

$$x = +\sqrt{26} - 3, x = -\sqrt{26} - 3$$

$$x \doteq 2.1, x \doteq 8.1$$

130. $-3(x+2)^2 = 27$

131. $-\frac{1}{4}(x-3)^2 = -6$

Challenge:

132. Solve. $x^2 + 20x - 6 = 0$

133. Determine the x-intercepts of
 $y = 2x^2 + 12x + 19$

Solve by completing the square, where possible. Express your answer answers as exact roots.

134. $x^2 + 20x - 6 = 0$

Solution:

Isolate the constant.

$$x^2 + 20x = 6$$

Complete the square.

$$(x^2 + 20x + [10^2]) = 6 + [10^2]$$

Factor.

$$(x + 10)^2 = 106$$

Square root both sides.

$$\sqrt{(x + 10)^2} = \pm\sqrt{106}$$

Isolate x.

$$x + 10 = \pm\sqrt{106}$$

$$x = -10 + \sqrt{106}, \quad x = -10 - \sqrt{106}$$

$$x \doteq 0.3 \quad x \doteq 20.3$$

135. $0 = x^2 - 8x + 10$

136. $x^2 + 16x - 7 = 0$

137. $0 = x^2 - 5x + 14$

138. $x^2 - 9x = -20$

139. $0 = x^2 + 24x - 100$

Determine the x-intercepts if they exist. Round your answer to the nearest tenth if needed.

$$140. y = 2x^2 + 12x + 19$$

To find the x-intercepts set $y=0$ and complete the square.

$$0 = 2x^2 + 12x + 19$$

$$0 = 2(x^2 + 6x) + 19$$

$$0 = 2(x^2 + 6x + 9) + 19 - 18$$

$$0 = 2(x+3)^2 + 1$$

We know there are NO x-intercepts since the parabola opens up and has a positive minimum value. Let's suppose you did not notice that. Solve as follows:

$$0 = 2(x+3)^2 + 1$$

Subtract 1 and divide by 2.

$$(x+3)^2 = -\frac{1}{2}$$

$$\sqrt{(x+3)^2} = \pm\sqrt{-\frac{1}{2}}$$

There are no x-intercepts since there are no real solutions to the square root of a negative number.

$$141. y = 4x^2 + 4x - 7$$

$$142. y = x^2 - 4x + 44$$

Challenge: Round your answer to the nearest tenth.

143. The world record blob jump sent a man 55 feet into the air. He followed the following parabolic

$$\text{pathway } v = -\frac{1}{3}(h-12)^2 + 55 \text{ where } v \text{ is vertical}$$

height in feet and h is the horizontal distance travelled. Determine the horizontal distance travelled when the man lands in the water.

144. A rectangular parking lot has a perimeter of 200 m and an area of 1300m^2 . Determine a quadratic equation that models this situation.

145. Determine the dimensions of the parking lot.

Round your answer to the nearest tenth.

146. The world record blob jump sent a man 55 feet into the air. He followed the following parabolic pathway $v = -\frac{1}{3}(h-12)^2 + 55$ where v is vertical height in feet and h horizontal distance travelled. Determine the horizontal distance travelled when the man hit the water. Round your answer to the nearest tenth.

Solution:

When the man hits the water $v=0$.

$$0 = -\frac{1}{3}(h-12)^2 + 55$$

$$-55 = -\frac{1}{3}(h-12)^2$$

$$165 = (h-12)^2$$

$$\pm\sqrt{165} = \sqrt{(h-12)^2}$$

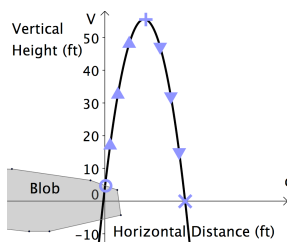
$$\pm\sqrt{165} = h-12$$

$$h = 12 \pm \sqrt{165}$$

$$h = 12 + \sqrt{165}, \quad h = 12 - \sqrt{165}$$

$$h \doteq 24.845, \quad h \doteq -0.845$$

The h -intercept is about 24.845 feet. The man travelled a distance of approximately 24.8 feet.



147. A circus performer is launched from a catapult from a height of 2 meters. The performer follows a parabolic path with $h = -\frac{1}{4}t^2 + t + 2$, where h is height in meters, and t is time in seconds. Determine how long the clown is in the air for?

148. A rectangular parking lot has a perimeter of 200 m and an area of 1300m². Determine a quadratic equation that models this situation.

Solution:

$$p = 2w + 2l = 200$$

$$w + l = 100$$

$$l = 100 - w$$

$$Area = 1300 = lw$$

$$Area = 1300 = (100 - w)w$$

$$1300 = 100w - w^2$$

$$0 = w^2 - 100w + 1300$$

Determine the dimensions of the parking lot.

$$0 = w^2 - 100w + 1300$$

Complete the square & solve for w .

$$1200 = (w - 50)^2$$

$$\pm\sqrt{1200} = \sqrt{(w - 50)^2}$$

$$\pm\sqrt{1200} = w - 50$$

$$50 \pm \sqrt{1200} = w$$

$$w \doteq 84.641, \quad w \doteq 15.359$$

There are two possible widths.

If $w=84.641$, the length can be found by

$$1300 \div 84.641 = 15.359.$$

149. A rectangular playground 8 m by 11 m will have a cement walkway of uniform width surrounding it. The combined area of the playground and the pathway is 190 m². Algebraically determine the outside dimensions of the frame.

Round your answer to the nearest tenth.

150. Jason launches a pebble into the sea using his slingshot. The pebble follows a parabolic pathway $h = -7t^2 + 21t + 5$ where h is height in meters and t is time in seconds. How long does it take for the pebble to land in the water.

151. A ball is thrown directly upward off a 200-foot cliff with an initial velocity of 96 feet per second. The ball's height in relation to time follows the pathway $h = -16t^2 + 96t + 200$, where h is the height in feet and t is time in seconds. How long does it take for the ball to descend to 50 feet above the ground?

Solve by completing the square, where possible. Express your answer answers accurate to one decimal place.

152. $-2x^2 - 4x + 1 = 0$

153. $0 = \frac{1}{2}x^2 - 4x + 7$

154. $0 = x^2 + 24x - 100$

Determine the x-intercepts if they exist. Round your answer to the nearest tenth where needed.

155. $y = -3(x + 21)^2 - 1$

156. $y = -2(x - 7)^2 + 9$

157. $y = \frac{1}{2}(x - 10)^2 + 11$

Challenge: Solve $ax^2 + bx + c = 0$ for x. Use the example on the left to help you solve the problem on the right.

158. Solve $3x^2 + 5x - 7 = 0$ by completing the square.

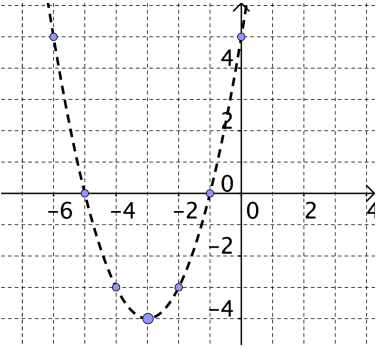
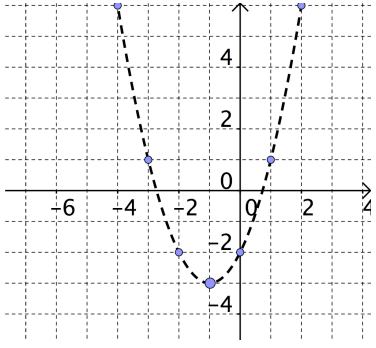
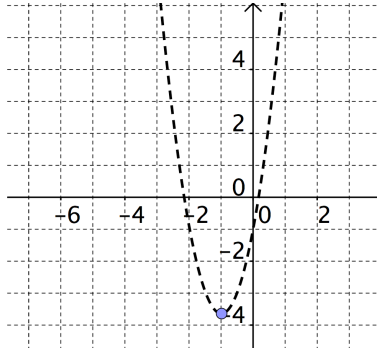
159. Solve $ax^2 + bx + c = 0$ for x, by completing the square.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula is a very important formula in mathematics. It provides another way of determining solutions to quadratic equations. It can solve any equation in the form $ax^2 + bx + c$, $a \neq 0$ quickly and efficiently.

Read each column.

$y = (x+3)^2 - 4$ <p>160. OR</p> $y = x^2 + 6x + 5$ 	$y = (x+1)^2 - 3$ <p>161. OR</p> $y = x^2 + 2x - 2$ 	$y = 2.7(x + 0.9889)^2 - 3.6403$ <p>162. OR</p> $y = 2.7x^2 + 5.34x - 1$ 
<p>We can see that $x^2 + 6x + 5 = 0$ can be solved by factoring.</p> $x^2 + 6x + 5 = 0$ $(x+5)(x+1) = 0$ $x = -5 \text{ or } x = -1$ <p>I have read this column</p> <p style="text-align: center;">Y or N</p>	<p>We can see that $x^2 + 2x - 2 = 0$ can be solved by completing the square.</p> $x^2 + 2x - 2 = 0$ $x^2 + 2x = 2$ $(x^2 + 2x + [1]) = 2 + [1]$ $(x+1)^2 = 3$ $\sqrt{(x+1)^2} = \pm\sqrt{3}$ $x+1 = \pm\sqrt{3}$ $x = -1 + \sqrt{3}, x = -1 - \sqrt{3}$ <p>I have read this column</p> <p style="text-align: center;">Y or N</p>	<p>We would likely solve $2.7x^2 + 5.34x - 1 = 0$, by completing the square. However, there is a better way to solve this.</p> <p>This type of question can be easily solved using the quadratic formula.</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>I have read this column</p> <p style="text-align: center;">Y or N</p>

Quadratic Formula

The quadratic formula states that if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$.

- A derivation of this formula is on the next page.
- For now we will focus on how it works.

Challenge: Solve using the quadratic formula.

163. Solve $x^2 + 5x + 6 = 0$. Exact answers only.

Name the a, b & c values:

$$a = \underline{\quad}, b = \underline{\quad}, c = \underline{\quad}$$

Substitute a,b & c into:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ and solve for } x.$$

164. Solve $x^2 + 2x - 2 = 0$. Exact answers only.

165. Solve $2.7x^2 + 5.34x - 1 = 0$. Round your answers to the nearest tenth.

Name the a, b & c values and solve for x.

166. $x^2 + 5x + 6 = 0$

Name the a, b & c values:

$$a = 1, b = 5, c = 6$$

Substitute a,b & c into:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ and solve for } x.$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{-5 \pm 1}{2} \rightarrow x = \frac{-5 + 1}{2}, x = \frac{-5 - 1}{2}$$

$$x = -2 \text{ OR } x = -3$$

167. $5x^2 - 2x - 6 = 0$

168. $-x^2 + x + 12 = 0$

The Derivation of the Quadratic Formula

Solve for x by completing the square.

169. $3x^2 + 5x - 7 = 0$		170. $ax^2 + bx + c = 0$
$3x^2 + 5x = 7$	Isolate the constant	$ax^2 + bx = -c$
$3\left(x^2 + \frac{5x}{3}\right) = 7$	Factor out the a-value on the left side.	$a\left(x^2 + \frac{bx}{a}\right) = -c$
$3\left(x^2 + \frac{5x}{3} + \left[\frac{5}{6}\right]^2\right) = 7 + 3\left[\frac{5}{6}\right]^2$	Complete the square and add it to both sides of the equation. (don't forget to multiply by the a value on the right side.)	$a\left(x^2 + \frac{bx}{a} + \left[\frac{b}{2a}\right]^2\right) = -c + a\left[\frac{b}{2a}\right]^2$
$3\left(x + \frac{5}{6}\right)^2 = 7 + 3\left[\frac{5}{6}\right]^2$	Factor the perfect square	$a\left(x + \frac{b}{2a}\right)^2 = -c + a\left[\frac{b}{2a}\right]^2$
$3\left(x + \frac{5}{6}\right)^2 = 7 + 3\left[\frac{25}{36}\right]$	Simplify the right side and combine fractions by creating a common denominator.	$a\left(x + \frac{b}{2a}\right)^2 = -c + a\left[\frac{b^2}{4a^2}\right]$
$3\left(x + \frac{5}{6}\right)^2 = \frac{84}{12} + \frac{25}{12}$		$a\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a} + \frac{b^2}{4a}$
$3\left(x + \frac{5}{6}\right)^2 = \frac{25 + 84}{12}$		$a\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a}$
$\left(x + \frac{5}{6}\right)^2 = \frac{109}{36}$	Simplify the fraction	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
$\sqrt{\left(x + \frac{5}{6}\right)^2} = \pm\sqrt{\frac{109}{36}}$	Square root both sides and simplify	$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$
$x + \frac{5}{6} = \pm\frac{\sqrt{109}}{6}$		$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$
$x = -\frac{5}{6} \pm \frac{\sqrt{109}}{6}$	Isolate x and combine fractions.	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-5 \pm \sqrt{109}}{6}$		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Challenge:

171. Solve $4m^2 - 2m = -2m^2 + 5m - 2$

Solve for m and round your answer to 3 decimals.

$$172. 4m^2 - 2m = -2m^2 + 5m - 2$$

Solution:

$$4m^2 - 2m = -2m^2 + 5m - 2$$

Add $2m^2$, subtract $5m$ and add 2 to both sides.

$$6m^2 - 7m + 2 = 0$$

Fill in the quadratic formula.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(2)}}{2(6)}$$

Simplify and solve for x.

$$x = \frac{7 \pm \sqrt{49 - 48}}{12}$$

$$x = \frac{7 \pm 1}{12} \rightarrow x = \frac{8}{12} \text{ or } x = \frac{6}{12}$$

$$x = \frac{2}{3} \text{ or } x = \frac{1}{2}$$

$$173. 4m^2 - 10m = 5m^2 - 2$$

$$174. m(3m - 7) = 1$$

Challenge: Round your answer to 3 decimals.

175. An NFL football field is rectangular in shape. It has an area of 57600ft^2 and a perimeter of 1040feet . Determine a quadratic equation that models this situation.

176. Determine the dimensions of the field.

177. A football is thrown and follows the flight pathway $h = -0.04d^2 + 0.85d + 6$, where h is height in meters and d is horizontal distance in meters. How far was the ball thrown if the ball was caught at a height of 5.5m ? Round your answer to the nearest tenth.

Round your answer to the nearest tenth.

178. The NFL football field is rectangular in shape. It has an area of 57600ft² and a perimeter of 1040feet. Determine a quadratic equation that models this situation.

Solution:

$$p = 2w + 2l = 1040 \quad \text{Area} = 57600 = lw$$

$$w + l = 520 \quad \text{Area} = 57600 = (520 - w)w$$

$$l = 520 - w \quad 57600 = 520w - w^2$$

$$0 = w^2 - 520w + 57600$$

Determine the dimensions of the field.

$$0 = w^2 - 520w + 57600$$

Use the quadratic formula

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-520) \pm \sqrt{(-520)^2 - 4(1)(57600)}}{2(1)}$$

$$w = \frac{520 \pm \sqrt{40000}}{2}$$

$$w = \frac{520 \pm 200}{2}, \therefore w = 360, w = 160$$

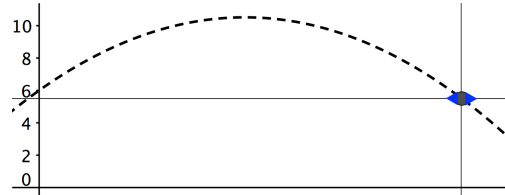
There are two possible widths.

If $w=360$, the length can be found by $57600 \div 360 = 160$.

180. A rectangular garden 6 m by 9 m will have a grass border of uniform width surrounding it. The combined area of the garden and the grass pathway is 80 m². Algebraically determine the outside dimensions of the frame.

179. A football is thrown and follows the flight pathway $h = -0.04d^2 + 0.85d + 6$, where h is height in meters and d is horizontal distance in meters. How far was the ball thrown if the ball was caught at a height of 5.5m? Round your answer to the nearest tenth.

Solution.



To find the time it takes for the ball to be caught at the height of 5.5, we need to substitute $h=5.5$ into

$$h = -0.04d^2 + 0.85d + 6$$

$$5.5 = -0.04d^2 + 0.85d + 6$$

$$0 = -0.04d^2 + 0.85d + 0.5$$

$$a = -0.04, b = 0.85, c = 0.5$$

$$w = \frac{-(0.85) \pm \sqrt{(0.85)^2 - 4(-0.04)(0.5)}}{2(-0.04)}$$

$$w = \frac{-0.85 \pm \sqrt{0.8025}}{-0.08},$$

$$\therefore w \doteq 21.823, \quad \cancel{w \doteq -0.573}$$

The ball was thrown 21.8 feet when it was caught.

181. A baseball player hits a ball into the air. The ball's height above the ground, in meters, t seconds after being hit, is approximated by $h(t) = 1 + 13t - 5t^2$. How long was the ball in the air for if the ball was caught at a height of 0.5 meters by a diving center fielder.

Round your answer to 1 decimal.

182. A potato is hurled into the air using a powerful slingshot. It is released at a height of 1 meter and follows a parabolic path with

$h = -20t^2 + 82t + 1$, where h is height in meters, and t is time in seconds. Determine how long the potato is in the air for?

183. An object is launched from ground level directly upward at 20 m/s and follows the parabolic pathway $h = -4.9t^2 + 20t$. How long was the object in the air for?

Solve for m to the nearest tenth, if possible.

184. $7m^2 + 5m = 5m^2 + 4m - 2$

185. $16m^2 + 17m = 10m^2 - 5$

186. $2m(3m - 5) = -10$

Review: Solve by any method.

$$187. 0 = 11m^2 + 10m - 1$$

$$188. 0 = 4x^2 - 20x$$

$$189. 0 = 25x^2 - 49$$

$$190. -4x(5x-1) = 0$$

$$191. 0 = (3x-2)^2 - 16$$

$$192. 0 = x^2 - 9x + 8$$

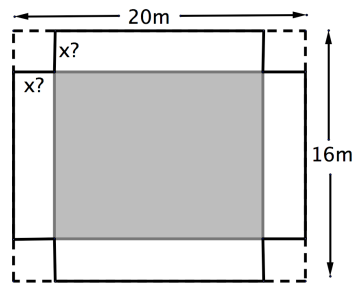
$$193. 0 = 36x^2 - 25x$$

$$194. 0 = 9(x-1)^2 - (2x)^2$$

$$195. 0 = 2(5x)^2 + 3(5x) + 1$$

Challenge: Round your answer to 1 decimal.

An open-topped box is to be made from a sheet of cardboard 16 in. by 20 in. The sides of the box are formed when a congruent square is cut from each corner, which allows the sides to fold up. The base of the box has an area of 165 sq. in.



196. Write an equation to represent the area of dimensions of the base.

197. Determine the side length of the congruent squares.

198. Determine the dimensions of the box

199. The diagonal of a rectangle is 73cm long. Determine the dimensions of the rectangle if the width is 7cm longer than the length.

Round your answer to 1 decimal.

200. An open-topped box is to be made from a sheet of cardboard 16 in. by 20 in. The sides of the box are formed when a congruent square is cut from each corner, which allows the sides to fold up. The base of the box has an area of 165 sq. in.

A. Write an equation to represent the area of dimensions of the base.

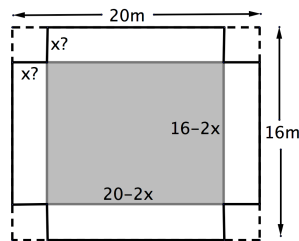
$$\text{Area} = lw$$

$$165 = (20 - 2x)(16 - 2x)$$

$$165 = (20 - 2x)(16 - 2x)$$

$$165 = 4x^2 - 72x + 320$$

$$0 = 4x^2 - 72x + 155$$



B. Determine the side length of the congruent squares.

$$0 = 4x^2 - 72x + 155$$

$$a = 4, b = -72, c = 155$$

$$x = \frac{-(-72) \pm \sqrt{(-72)^2 - 4(4)(155)}}{2(4)} = \frac{72 \pm \sqrt{2704}}{8}$$

$$x = \frac{72 \pm 52}{8} \therefore x = 15.5 \text{ \& } x = 2.5$$

15.5 is rejected because $20 - 2(15.5) = -11$

C. Determine the dimensions of the box

$$l = 20 - 2x \quad w = 16 - 2x$$

$$l = 20 - 2(2.5) \text{ \& } w = 16 - 2(2.5)$$

$$l = 15 \quad w = 11$$

201. An open-topped storage box is to be made from a sheet of cardboard 8 feet by 10 feet. The sides of the box are formed when a congruent square is cut from each corner, which allows the sides to fold up. The base of the box has an area of 28.16 sq. feet.

A. Write an equation to represent the area of dimensions of the base.

B. Determine the side length of the congruent squares.

C. Determine the dimensions of the box.

202. The diagonal of a rectangle is 73cm long. Determine the dimensions of the rectangle if the width is 7cm longer than the length.

Solution.

$$h^2 = w^2 + l^2$$

$$73^2 = x^2 + (x+7)^2$$

$$5329 = x^2 + x^2 + 14x + 49$$

$$0 = 2x^2 + 14x - 5280$$

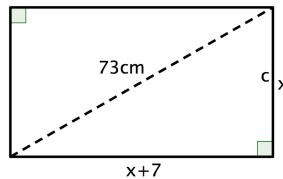
$$0 = x^2 + 7x - 2640$$

$$a = 1, b = 7, c = -2640$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2640)}}{2(1)} = \frac{-7 \pm \sqrt{10609}}{2}$$

$$= \frac{-7 \pm 103}{2} \therefore x = -55 \text{ or } x = 48$$

The dimensions are 73, 48 and $48+7=55$



Round your answer to 3 decimals.

204. Jordan and Sarah both begin at the same point. Jordan drives her car directly south at 80km/hour, while Sara rides her bike east at 25km/hour.

- A. Write an equation to represent their distance.
- B. How long will it take before they are 200km apart?

205. John and Aaron start running from the same point. John runs due north at a rate of 10km/hour and Aaron runs due west at a rate of 12km/hour.

- A. Write an equation to represent their distance.
- B. How long will it take for the two men to be 80km apart?

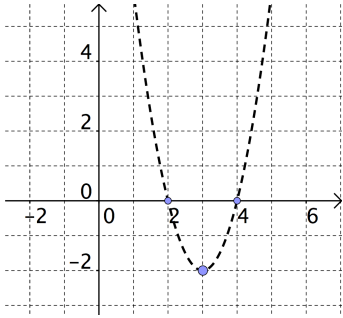
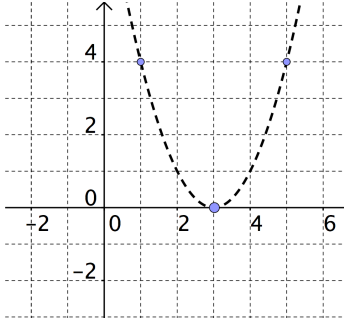
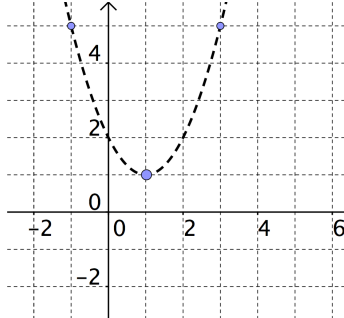
206. 260 feet of flagging tape is to be used to mark of a right triangular area in a field. If the hypotenuse is 109 feet long determine the length of the other two sides.

207. A small paper box is to be made from a sheet of paper measuring 20cm by 12 cm. The sides of the box are formed when a congruent square is cut from each corner, which allows the sides to fold up. The base of the box has an area of 65cm^2 .

- A. Write an equation to represent the area of dimensions of the base.
- B. Determine the side length of the congruent squares.
- C. Determine the dimensions of the box.

The Discriminant

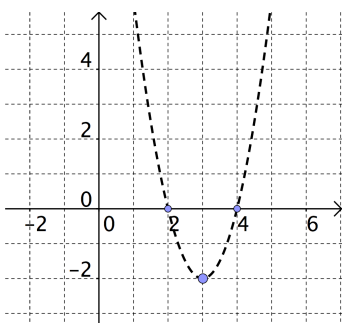
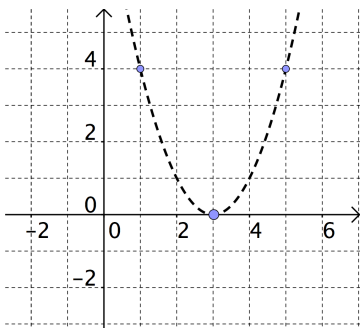
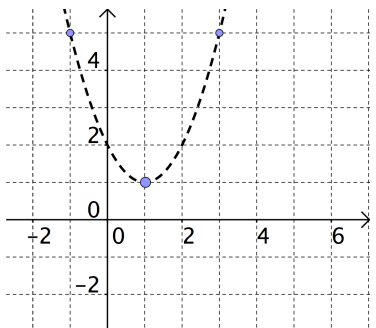
Read each column and answer the multiple-choice question.

<p>208. $y = 2x^2 - 12x + 16$ Turns into $y = 2(x - 3)^2 - 2$</p> 	<p>209. $y = x^2 - 6x + 9$ Turns into $y = (x - 3)^2$</p> 	<p>210. $y = x^2 - 2x + 2$ Turns into $y = (x - 1)^2 + 1$</p> 
<p>Two different x-intercepts. Determine the roots using the QF $0 = 2x^2 - 12x + 16$ $a = 2, b = -12, c = 16$</p> $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(16)}}{2(2)}$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = \frac{12 \pm \sqrt{16}}{4} *$ </div> $x = \frac{12 \pm 4}{4} \rightarrow x = 2 \text{ and } x = 4$	<p>One x-intercept. Determine the roots using the QF $0 = x^2 - 6x + 9$ $a = 1, b = -6, c = 9$</p> $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = \frac{6 \pm \sqrt{0}}{2} **$ </div> $x = \frac{6}{2} \rightarrow x = 3$	<p>No x-intercepts Determine the roots using the QF $0 = x^2 - 2x + 2$ $a = 1, b = -2, c = 2$</p> $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = \frac{2 \pm \sqrt{-4}}{2} ***$ </div> <p>→ No solution</p>
<p>211. *There will be two different real roots if: Circle one.</p> <p>A. $x = \frac{-b \pm \sqrt{+}}{2a}$</p> <p>B. $x = \frac{-b \pm \sqrt{0}}{2a}$</p> <p>C. $x = \frac{-b \pm \sqrt{-}}{2a}$</p>	<p>212. **There will be one equal real root if: Circle one.</p> <p>D. $x = \frac{-b \pm \sqrt{+}}{2a}$</p> <p>E. $x = \frac{-b \pm \sqrt{0}}{2a}$</p> <p>A. $x = \frac{-b \pm \sqrt{-}}{2a}$</p>	<p>213. ***There will be no real roots if: Circle one.</p> <p>F. $x = \frac{-b \pm \sqrt{+}}{2a}$</p> <p>G. $x = \frac{-b \pm \sqrt{0}}{2a}$</p> <p>A. $x = \frac{-b \pm \sqrt{-}}{2a}$</p>

Definitions:

Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	The formula is used to solve equations in this form $ax^2 + bx + c = 0$
Discriminant	$b^2 - 4ac$	Piece of QF under the square root sign
Nature of the roots	If $b^2 - 4ac > 0 \rightarrow$ Two different real roots. If $b^2 - 4ac = 0 \rightarrow$ One equal real root. If $b^2 - 4ac < 0 \rightarrow$ No real roots.	i.e. 2&4 or -2&7 i.e. 3&3 or -4&-4 i.e. No x-intercepts

Determine the value of the discriminant and the Nature of the roots.

<p>214. Graph. $y = 2x^2 - 12x + 16$</p> <p>Turns into $y = 2(x - 3)^2 - 2$</p> 	<p>215. Graph. $y = x^2 - 6x + 9$</p> <p>Turns into $y = (x - 3)^2$</p> 	<p>216. Graph. $y = x^2 - 2x + 2$</p> <p>Turns into $y = (x - 1)^2 + 1$</p> 
<p>A. State the value of the discriminant.</p> <p>Discriminant = $b^2 - 4ac$</p> <p>B. State the nature of the roots.</p>	<p>A. State the value of the discriminant.</p> <p>Discriminant = $b^2 - 4ac$</p> <p>B. State the nature of the roots.</p>	<p>A. State the value of the discriminant.</p> <p>Discriminant = $b^2 - 4ac$</p> <p>B. State the nature of the roots.</p>

Reminder.

<p>217. State the number of solutions if $b^2 - 4ac = 0$.</p> <p>Choose one:</p> <p>A. Two different real roots. B. One real root. C. No real roots.</p>	<p>218. State the number of solutions if $b^2 - 4ac > 0$.</p> <p>Choose one:</p> <p>A. Two different real roots. B. One real root. C. No real roots.</p>	<p>219. State the number of solutions if $b^2 - 4ac < 0$.</p> <p>Choose one:</p> <p>A. Two different real roots. B. One real root. C. No real roots.</p>
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Determine the value of the discriminant.

220. $y = 2x^2 + 5x + 10$

221. $y = 5x^2 + 30x + 45$

222. $y = 4x^2 - 3x - 5$

Determine the nature of the roots.

223. $y = 2x^2 + 5x + 10$

224. $y = 5x^2 + 30x + 45$

225. $y = 4x^2 - 3x - 5$

Determine an expression for the discriminant.

226. $2x^2 + kx - 8 = 0$

227. $kx^2 + 2x + 3 = 0$

228. $2x^2 + kx + k = 0$

Challenge:

229. Find the value(s) of K that leads to 2 different real roots.
 $x^2 - kx + 1 = 0$

230. Find the value(s) of K that leads to one real root.
 $(2k+1)x^2 + 8x + 6 = 0$

231. Find the value(s) of K that leads to no real roots.
 $kx^2 - 4x + k = 0$

Solutions

<p>232. Find the value(s) of K that leads to 2 different real roots. $x^2 - kx + 1 = 0$</p> <p>Possible solution strategy: Must satisfy $b^2 - 4ac > 0$</p> $(-k)^2 - 4(1)(1) > 0$ $k^2 - 4 > 0$ $k^2 > 4$ <p>So either $k > 2$ or $-k > 2$ $k < -2$</p> <p>Two 2 different real roots when</p> <p>$k > 2$ i.e. 2,1,3,4,5... $k < -2$ i.e. -2,1,-3,-4,-5...</p>	<p>233. Find the value(s) of K that leads to one real root. $(2k+1)x^2 + 8x + 6 = 0$</p> <p>Possible solution strategy: Must satisfy $b^2 - 4ac = 0$</p> $8^2 - 4(2k+1)(6) = 0$ $64 - 48k - 24 = 0$ $40 - 48k = 0$ $40 = 48k$ <p>So one real root when....</p> $k = \frac{40}{48} = \frac{5}{6}$	<p>234. Find the value(s) of K that leads to 2 no real roots. $kx^2 - 4x + k = 0$</p> <p>Possible solution strategy: Must Satisfy $b^2 - 4ac < 0$</p> $(-4)^2 - 4(k)(k) < 0$ $16 - 4k^2 < 0$ $-4k^2 < -16$ $4k^2 > 16$ $k^2 > 4$ <p>So either $k > 2$ or $-k > 2$ $k < -2$</p> <p>So 0 real roots when... $k > 2$ or $k < -2$</p>
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Find the value(s) of K that leads to the following number of solutions.

<p>235. Find the value(s) of K that leads to 2 different real roots. $x^2 - kx + 9 = 0$</p>	<p>236. Find the value(s) of K that leads to one real root. $(2k)x^2 + 8x + 6 = 0$</p>	<p>237. Find the value(s) of K that leads to no real roots. $kx^2 - 6x + k = 0$</p>
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How many unique x-intercepts do the following quadratic equations have?

238. $y = 4x^2 + 24x + 36$	239. $y = -x^2 + 2x - 5$	240. $y = x^2 - 7x - 4$
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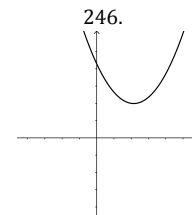
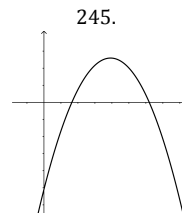
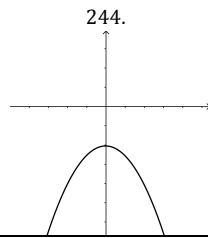
241. Find the value(s) of K that leads to 2 different real roots.
 $kx^2 + 4x - 3 = 0$

242. Find the value(s) of K that leads to one real root.
 $x^2 + kx + 7 = 0$

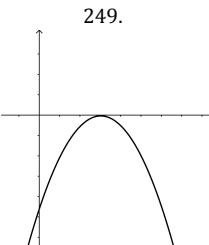
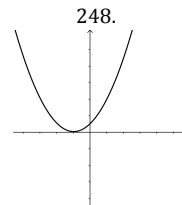
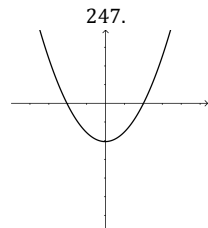
243. Find the value(s) of K that leads to 2 no real roots.
 $kx^2 - 8x + 9 = 0$

Match the letters with the pictures where appropriate.

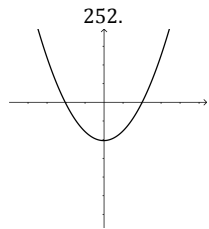
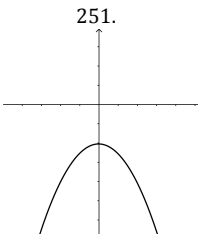
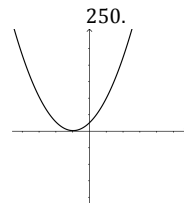
- A. Discriminant = 4
- B. Discriminant = -7
- C. Discriminant = 0



- A. $b^2 - 4ac > 0$
- B. $b^2 - 4ac = 0$
- C. $b^2 - 4ac < 0$



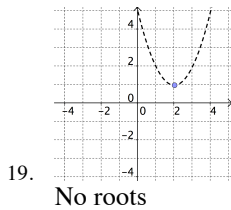
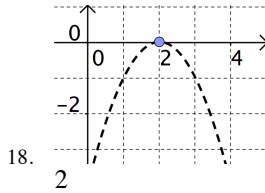
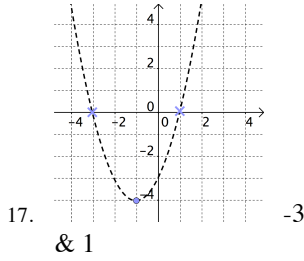
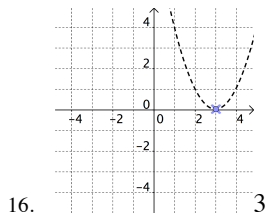
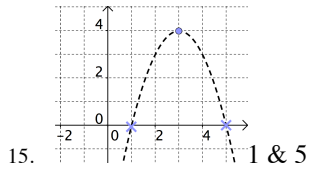
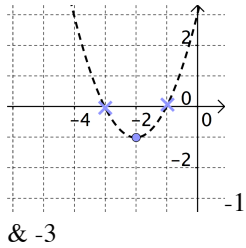
- A. 2 real roots
- B. 1 real root
- C. No real roots



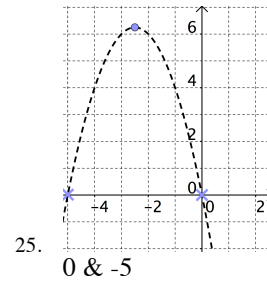
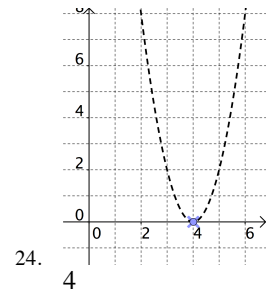
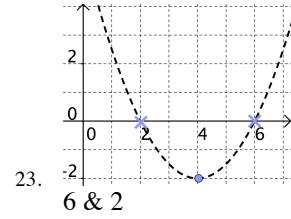
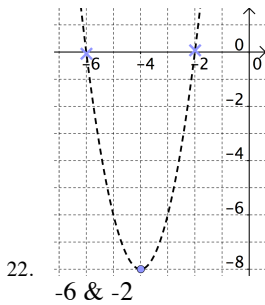
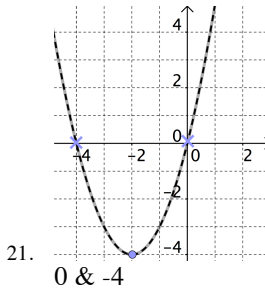
Answer Key

Please report any errors in the answer key to your teacher as soon as possible.

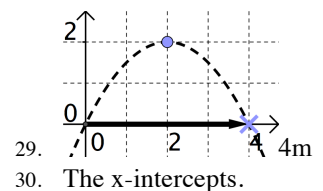
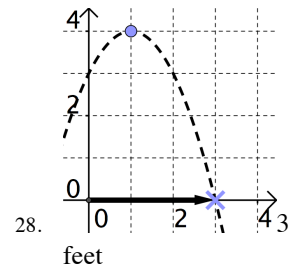
1. 70m
2. The x-intercepts
3. Zero
4. -4
5. 1 and 5
6. (1,0)
7. -3 and 1
8. No x-intercepts
9. 0
10. The zeros of $y = ax^2 + bx + c$ occur when $y=0$ which makes the zeros the same as the solutions to $0 = ax^2 + bx + c$.
11. 2
12. 1
13. 0

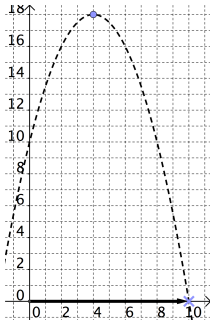


20. Answered on the page.

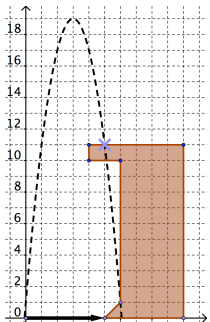


26. 6 seconds, the x-coordinate (0) of the y-intercept and the x-intercept (6)
27. 25 feet

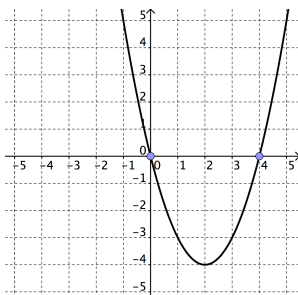




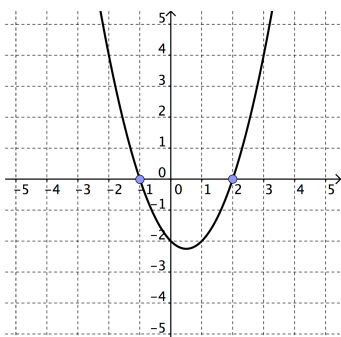
31. 10 seconds



32. 5 inches

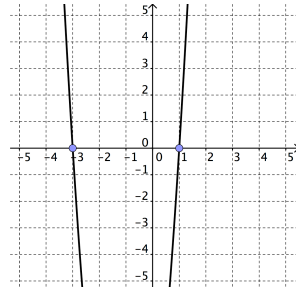


33. 0 & 4



34. -1 & 2 (This will be difficult to solve by graphing because the both values of the vertex are decimals.)

We will learn a new method in this section.)



35. -3 & 1 this will be difficult to solve by graphing because the vertex does not fit on the grid. In this section we will learn a new technique.

36. #34 is difficult to solve by graphing because the vertex is made up of decimals. Accuracy may be a factor. #35 is difficult to solve by graphing because the vertex does not fit on the grid. Accuracy may be a factor.

37. $(x+8)(x+2)$

38. $(5x+2)(x+3)$

39. $(2x+1)(x+5) - \frac{1}{2}$

40. 1 & 5

41. D

42. 4 & -2

43. D

44. -1 & 3

45. E

46. $Y=0$ at the x-intercept. So $y=(x-1)(x-5)=0$ at the x-intercept.

47. 0 & -5

48. $-\frac{1}{2}$

49. $-\frac{1}{2}$ & -5

50. Answered on the page.

51. $-\frac{3}{2}$ & 2

52. 0 & 2

53. Answered on the page.

54. -1 & $\frac{1}{2}$

55. $-\frac{1}{5}$ & $\frac{8}{3}$

56. $y=(x-5)(x+2)$

57. $y=(x+3)(x+1)$

58. $y=(5x-2)(x-4)$

59. $y=(2x+1)(3x-1)$

60. $y=(x+1)(x)$

61. $y=x(x+17)$

62. Answered on the next page.

63. Answered on the next page.

64. Answered on the next page.

65. Answered on the page.

66. -6 & 4

67. 0 & 2

68. Answered on the page.

69. 12 & -3

70. -9 & 5

71. Answered on the page.

72. $-\frac{1}{3}$ & $-\frac{5}{2}$

73. $-\frac{1}{2}$ & -5

74. 11 & 7

75. $\frac{3}{2}$ & 1

76. -2 & -5

77. -1 & $\frac{7}{3}$

78. 45 & 1

79. 0, -1 & $\frac{5}{2}$

80. $\pm \frac{5}{6}$

81. 2 & $-\frac{8}{3}$

82. -4 & -3

83. Answered on the page.

84. $\pm \frac{7}{11}$

85. $\pm \frac{10}{9}$

86. Answered on the page.

87. 0 & 1

88. $\frac{3}{4}$
89. Answered on the page.
90. 15 & 2
91. $4 \text{ \& } \frac{9}{2}$
92. Answered on the next page.
93. Answered on the next page.
94. Answered on the page.
95. Initial equation \rightarrow
 $(11+2x)(8+2x)=180$,
simplified $(2x+23)(x-2)=0$ & the solution is 2cm.
96. Answered on the page.
97. $-4.9(t-6)(t+2)=0$ & $t=6$
(-2 is not a solution)
98. Area=1200=(70-w)w
99. The dimensions are 30m by 40m.
100. $9=-5t^2+13t+1 \rightarrow 0=5t^2-13t+8 \rightarrow$ The ball could have been caught at two different times; 1s & $\frac{8}{5}$ s.
101. ± 5
102. 0 & 36
103. ± 8
104. 2 & -16
105. $-\frac{1}{6}$ & $-\frac{5}{2}$
106. $-\frac{13}{3}$ & $-\frac{41}{11}$
107. -2 & -6
108. -44 & 84. These numbers were found using methods other than factoring.
109. -1.763932 & -6.236068. These numbers were approximated using methods other than factoring.
110. #108 is difficult to solve because the numbers are so big that factoring will take a while to find the right numbers. #109 is difficult to solve because the solutions are not whole numbers.
111. Answered on the page.
112. $(x+5)^2 - 5$
113. $10(x+1)^2 - 7$
114. ± 2
115. -9 & -5
116. 2.099 & -8.099
117. Answered on the page.
118. ± 5
119. No solution.
120. $\pm\sqrt{26}$
121. No solution.
122. $\pm 2\sqrt{6}$
123. $\pm\sqrt{2}$
124. $\pm 2\sqrt{3}$
125. No solution.
126. Answered on the page.
127. 0 & 10
128. 6 & -8
129. Answered on the page.
130. No solution.
131. 7.9 & -1.9
132. Answered on the next page.
133. Answered on the next page.
134. Answered on the page.
135. $4 + \sqrt{6}$ & $4 - \sqrt{6}$
136. $-8 + \sqrt{71}$ & $-8 - \sqrt{71}$
137. No solution.
138. 4 & 5
139. $-12 + 2\sqrt{61}$ & $-12 - 2\sqrt{61}$
140. Answered on the page.
141. 0.9 & -1.9
142. No solution.
143. Answered on the next page.
144. Answered on the next page.
145. Answered on the next page.
146. Answered on the page.
147. 5.5s
148. Answered on the page.
149. 12.4m by 15.4m
150. 3.2s
151. 7.3s
152. -2.2 & 0.2
153. 5.4 & 2.6
154. -27.6 & 3.6
155. No solution.
156. 9.1 & 4.9
157. No solution.
158. Answered at #169.
159. Answered at #170
160. Make sure you read the column ☺.
161. Make sure you read the column ☺.
162. Make sure you read the column ☺.
163. Answered at #166.
164. $a=1$, $b=2$ & $c=-2$ and
 $x = -1 + \sqrt{3}$
& $x = -1 - \sqrt{3}$
165. $a=2.7$, $b=5.34$ & $c=-1$
and $x=0.2$ & $x=-2.2$.
166. Answered on the page.
167. 5, -2, -6 and
 $x=1.314 \rightarrow 1.3$ &
 $x = -0.914 \rightarrow -0.9$
168. 4 & -3
169. Answered on the page.
170. Answered on the page.
171. Answered on the next page.
172. Answered on the page.
173. 0.196 & -10.196
174. 2.468 & -0.135
175. Answered on the next page.
176. Answered on the next page.
177. Answered on the next page.
178. Answered on the page.
179. Answered on the page.
180. $(6+2x)(9+2x)=80 \rightarrow 2x^2 + 15x - 13 = 0 \rightarrow x=0.785$.
Dimensions 7.56m by 9.56m
181. $0.5=1+13t-5t^2 \rightarrow 0=-5t^2+13t+0.5 \rightarrow x=2.638$
182. 4.1s
183. 4.1s
184. No Solution.
185. $-\frac{1}{3}$ & $-\frac{5}{2}$
186. No Solution.
187. $\frac{1}{11}$ & -1
188. 0 & 5

189. $\pm \frac{7}{5}$

190. $0 \text{ \& } \frac{1}{5}$

191. $2 \text{ \& } -\frac{2}{3}$

192. $8 \text{ \& } 1$

193. $0 \text{ \& } \frac{25}{36}$

194. $3 \text{ \& } \frac{3}{5}$

195. $-\frac{1}{10} \text{ \& } -\frac{1}{5}$

196. $(20-2x)(16-2x)=165$
 $\rightarrow 4x^2-72x+155=0$

197. 2.5 is the only solution.
15.5 is too big.

198. Answered on the next page.

199. Answered on the next page.

200. Answered on the page.

201. $(8-2x)(10-2x)=28.16 \rightarrow 4x^2-36x+51.84=0$, 1.8, 4.4
by 6.4

202. Answered on the page.

203. 7, 24 & 25

204.

$$d^2 = (80h)^2 + (25h)^2$$
$$\rightarrow$$

$$200^2 = (80h)^2 + (25h)^2$$
$$\rightarrow 2.386h.$$

205.

$$d^2 = (10h)^2 + (12h)^2$$

 \rightarrow

$$80^2 = (10h)^2 + (12h)^2$$
$$\rightarrow 5.121.$$

206. 60,91 and 109 (The equation is $109^2=x^2+(151-x)^2$)207. $(20-2x)(12-2x)=65 \rightarrow 4x^2-64x+175$,
3.5 and 13cm by 5cm.

208. Make sure you read the column ☺.

209. Make sure you read the column ☺.

210. Make sure you read the column ☺.

211. A

212. B

213. C

214. 16 and 2 real roots

215. 0 and 1 real root

216. -4 and no real roots

217. B

218. A

219. C

220. -55

221. 0

222. 89

223. No real roots

224. 1 real root

225. 2 real roots

226. k^2+64 227. $4-12k$ 228. k^2-8k

229. Answered on the next page.

230. Answered on the next page.

231. Answered on the next page.

232. Answered on the page.

233. Answered on the page.

234. Answered on the page.

235. $k>6 \text{ \& } k<-6$

236. $\frac{4}{3}$

237. $k>3 \text{ \& } k<-3$

238. 1

239. 0

240. 2

241. $k > -\frac{4}{3}$

242. $k = 2\sqrt{7} \text{ \& } k = -2\sqrt{7}$

243. $k > \frac{16}{9}$

244. B

245. A

246. B

247. A

248. B

249. B

250. B

251. C

252. A